4 Pricing of Options (Valuation of Options)

Option is an uneven contract. It gives right to only one of the parties. It gives right to buyer or holder. It binds the other party, i.e., the writer or seller to a contract. The writer or seller has obligation to perform the contract. He has to accept the decision of the buyer. Thus, the buyer or holder enjoys an advantageous position. So he has to pay a price for enjoying the right to buy or sell. The amount that is paid by the buyer of the option to the seller is called premium.

Meaning of Option Price

The amount which is paid by the option buyer to the option seller is the price of an option contract. In fact, the option price or option value is the premium paid by the option buyer to the option seller. The price of an option contract is computed by the demand and supply of the underlying asset.

Intrinsic Value and Time Value

There is always a choice available to the holder – whether to exercise immediately or defer it for later point of time. Therefore, the price of an option or an option premium is made up of two components – the intrinsic value and the time value. Thus,

Option premium (or option price) = Intrinsic value + Time value

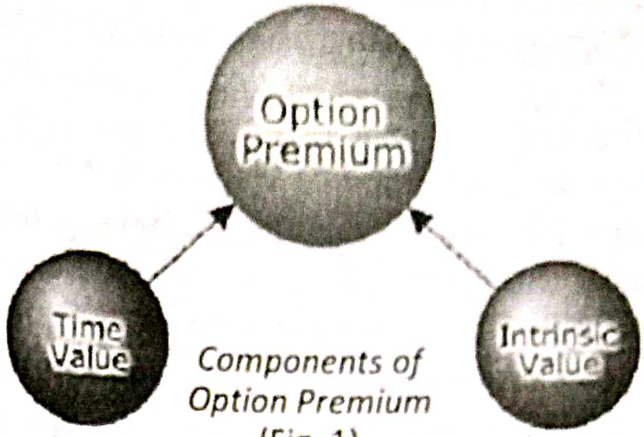
Intrinsic Value

Literally speaking, intrinsic value means the value contained in the asset itself. The intrinsic value is the amount that the option buyer would receive if the option is exercised. It is the value of the option if it is exercised immediately. In case of call option, intrinsic value is spot price less the strike price of the underlying asset. In case of put option, it is the strike price less the spot price of the underlying asset. In short, the intrinsic value of a call is the amount by which an option is in-the-money (the buyer will exercise it only when it is in-the-money). An option that is out-of-the-money (if the strike price of the call option is greater than the spot price) will have zero intrinsic value. Similarly, an option that is at-the-money (strike price is equal to the spot price) will have zero intrinsic value. Thus, only in-the-money options have intrinsic value.

The intrinsic value is:

For call option = Max (S – E, 0)

For put option = Max (E – S, 0)



Components of Option Premium
(Fig. 1)

This means that intrinsic value of a call option is greater of zero or (S - E). Similarly, intrinsic value of the put option is the greater of zero or (E - S). This further means that the intrinsic value of an option can never be negative. This is because the option holder (buyer) cannot be forced to exercise it. Intrinsic value is also known as *parity value* or *fundamental value* or *underlying value.*

For example, if the spot price (cash price) of an asset is ₹ 50, any call option with a strike price less than ₹ 50 will be in-the-money or will have intrinsic value. But any put option with a strike price less than ₹ 50 will be out- of- the money or will not have any intrinsic value. On the other hand, any call option with a strike price that is more than ₹ 50 will not have any intrinsic value. But a put option with a strike price higher than ₹ 50 will be in-the-money or will have intrinsic value.

In short: In case of call option, IV = Stock or market price - Strike price

In case of put option, IV = Strike price - Stock or market price

Time Value

It is profitable for the holder of an in-the-money option to wait rather than exercise it immediately, because by waiting he may be able to get higher profit in the future. For example, the holder of a call option which is in the money can realize the intrinsic value (S - E) immediately by exercising it. But, if he waits, there is a probability that the share price may increase further in the future (i.e., S may increase further) and he may be able to get a higher profit then. Thus, waiting for the future has a potentiality of increasing the profit expected from an option contract. The option is then said to have time value.

Time value is the difference between the option premium and the intrinsic value. It is the amount by which the market price of an option exceeds its intrinsic value. For the European option, this argument does not hold. Time value is also called *extrinsic value* or *premium over parity* or *speculative value* or *volatile value*. It is expressed as below :

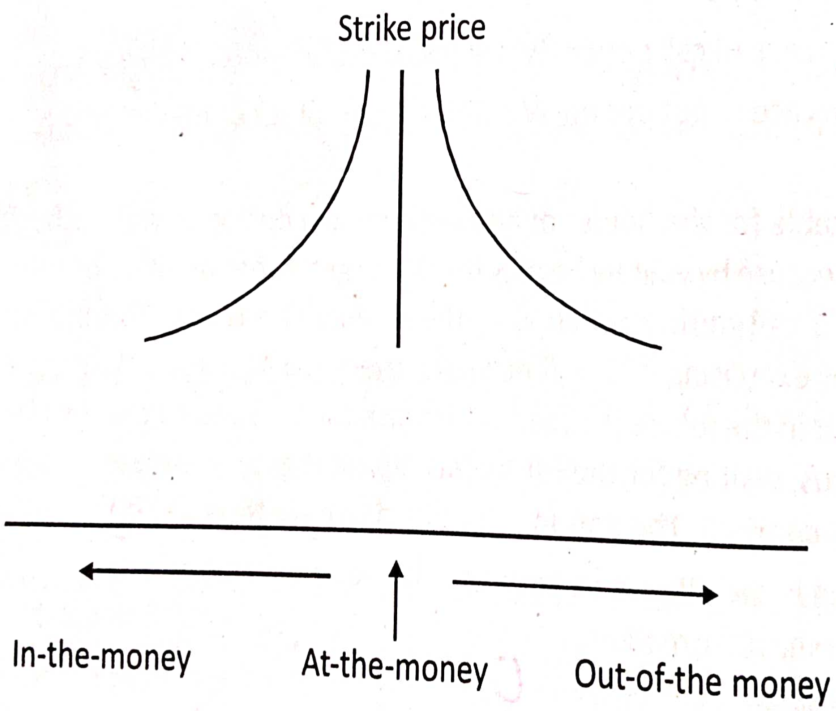
Time value = Option premium - Intrinsic value

Thus, a call that is out-of-the-money or at-the-money has only time value (i.e., no intrinsic value). Usually, the maximum time value exists when the call (or put) is at the money. Both calls and puts have time value.

Time value depends on time to expiration of the option and volatility in the prices of the underlying asset. It may be noted that in case of out-of-the-money option or at-the-money option, the entire premium paid by the option buyer to the option seller is the time value of the option. Time value of an option cannot be negative. Further, the more time that remains for the expiration, the higher the time value tends to be. For example, if an option with nine months remaining until expiration will have a higher time value than an option with the same strike price with less expiration time like six months, three months, one month etc. As the option approaches its maturity, time value will be diminishing and will reach to zero. At expiration, the option is worth only its intrinsic value (i.e., zero time value). This is why option is referred to as "a wasting asset". It may be noted that if an option is out of money, its intrinsic value will be zero and the option value, if any, is only time value.

At expiration, there is no question of waiting (no waiting). Hence, time value is zero at expiration. In other words, at expiration, no time remains in the life of an option. Therefore, there is no time value at expiration.

The time value of an option also depends upon whether option is in-the-money, at-the-money or out-of-the money. Time value is the maximum for at-the-money options due to the high uncertainty about the future movement of the price of underlying asset. As the option goes deeper in-the-money or out-of-the money, as a result of uncertainty with regard to the price of underlying asset, the time value of option diminishes. This is shown in the following diagram:



Time Value of Options (Fig. 2)

Option Formula

1. Premium = Time Value + Intrinsic Value
2. Intrinsic Value (Call) = Max. (0, Spot - Strike)
3. Intrinsic Value (Put) = Max. (0, Strike - Spot)
4. Time Value is Max. at the start of the contract
5. Time Value = 0 at Expiry

Example 1

The current market price of the shares of XYZ Ltd. is ₹ 27.50. A call option is available at a strike price of ₹ 25 and a put option is available at a strike price of ₹ 30. Find out the intrinsic value of both the options. If the options are available at a premium of ₹ 4 and ₹ 3 respectively, find out the time value of option.

Solution

Call option : Intrinsic Value = Market Price - E = ₹ 27.50 - ₹ 25 = ₹ 2.50

Time Value = Premium - Intrinsic Value

= ₹ 4 - 2.50 = ₹ 1.50

Put Option : Intrisic Value = Strike Price - Market Price = ₹ 30 - ₹ 27.50

= ₹ 2.50

Time Value = Premium - Intrinsic Value

= ₹ 3 - 2.50 = ₹ 0.50

Example 2

Shares of Akash Ltd. are selling at ₹3000. Following options are available for one month duration.

Call Options		Put Options	
Strike Price	Premium	Strike Price	Premium
₹2,900	₹ 120	₹3,100	₹ 125
3,000	35	3,000	40
3,100	5	2,900	10

Find out the Intrinsic Value and Time Value of the Call and Put options.

Solution

For Call options and Put options, the Intrinsic Value (IV) and Time Value (TV) can be found as follows :

Call options				Put options			
Strike Price	Premium (Pm.)	IV = (S - E)	TV = (Pm. - IV)	Strike Price	Premium (Pm.)	IV= (E - S)	TV = (Pm. - IV)
₹2,900	₹ 120	₹ 100	₹ 20	₹ 3,100	₹ 125	₹ 100	₹ 25
3,000	35	0	35	3,000	40	0	40
3,100	5	0	5	2,900	10	0	10

It may be noted that if an option is out of money, its intrinsic value is zero and the option value, if any, is only time value.

Factors Affecting Option Price (Determinants of Option Pricing)

Like the price of any other product, the price (premium) of an option is determined by demand and supply factors in the market. Option premium (price) is a combination of intrinsic value and time value. Intrinsic value in turn is a function of the difference between the strike price and the market price of the underlying asset. Time value is a function of volatility of the price of underlying asset, time to the expiration of the option and prevailing interest rate in the economy. The following are the factors affecting the price of an option.

1. **Current price or price of the underlying (S):** When the price of the underlying or current price of the asset changes, the price or value of the option also changes. When a call option is exercised the pay-off resulting there from equals the excess of the stock or asset price over the exercise price. The price of the call option is determined by the differential of S and E. Higher the asset price higher the differential and hence the price or value of the call. This means that a call option will be more valuable when stock price increases and less valuable when it decreases. In other words, for a call option the option price rises as the stock price increases and vice versa. As the current stock price goes up, the higher is the probability that the call will be in- the- money. As a result, the call price will increase. Most investors buy a call option to get benefit from the price rise. Therefore, investors must pay a higher premium if the asset price increases. A call option becomes more expensive as writer of the call assumes greater risk with rising spot price.

On the other hand, for a put option, the pay-off or the intrinsic value is the difference between exercise price and the stock price. As such, the higher the stock price for a given exercise price, the lower will be the price of the option. As the spot price increases, the differential E – S decreases. Therefore, the value of put falls with increase in the price of the asset. Investors buy put option to protect themselves against the fall in the prices. As the price goes up, the put holder is less likely to exercise his option. It is a favourable situation for writer of the put. Therefore, he can be induced to write the put option with lesser premium.

2. **Exercise Price (E):** The impact of exercise price (or strike price) on the value of the option is opposite to that of the spot price. Higher the exercise price lower is the value of the call option. As the exercise price goes up the chances that the share price will exceed it become lesser and lesser. With increase in the exercise price the call option becomes more and more out-of-the-money. The value or price of the call is determined by differential of spot price and the exercise price. With the increase in the exercise price, the price of the call falls. For example, suppose, two calls on a stock with identical expiry dates have exercise prices of ₹ 90 and ₹ 95 respectively. The holder of the former call can buy the underlying security at ₹ 90, and can be in the same position as the holder of the other call who could buy the same security at ₹ 95, plus the cash left over. Thus, the call with a higher exercise price cannot be valued higher. For a put, the effect will be in the opposite direction. A higher exercise price means that there is higher probability that the put will be in-the-money. So the put price increases as the exercise price increases.

3. **Time to expiration (T) :** Generally both calls and puts will benefit from increased time left for expiration. The reason is that there is more time for a big move in the stock price. With more time available there would be greater chances of achieving the exercise price. This general statement holds true for both call and put options. The option with longer life will be valued more than the option with shorter life. There are greater chances for price going above E (for call options) or falling below E (for put option). Conversely, lesser time available will make options less valuable. Irrespective of whether it is a call option or a put option, the chances that option will turn in-the-money increased with increased availability of option period. Therefore, a call/put of three months will cost more than the call/put with expiry in one month.

4. **Volatility or Variability of the price of the stock (σ):** A major factor affecting the price of options is volatility or variability of price. Volatility is the degree to which price of a stock or stock index or asset tends to fluctuate over time. It represents the uncertainty attached to the future movement of stock price. When the underlying asset becomes more volatile, the price of both call and put options will increase. As volatility increases, the probability that the stock will do very well or very bad increases. The owner of the call benefits from the price increase but his downward risk is limited because the maximum he loses is the option premium. Similarly, the owner of a put option benefits from decrease in the stock price. His risk in case of adverse, upward price movement is limited. Thus the values of both calls and puts increase with increase in volatility. The buyer of the option receives full benefit of favourable outcomes but avoids the unfavourable ones. As higher volatility increases the chances of an option going in-the-money at any point of time during the life of the contract, it increases the risk to the option seller. As a result, higher volatility makes the option (both call and put) more expensive (more price).

5. **Interest rate (r) :** Interest rates also affect the option price. The higher the interest rate the lower the present value of the exercise price. As a result, the value of the call will increase and the value of the put will decrease. The decrease in the present value of the exercise price will adversely affect the price of the put option. All other factors remaining constant, the higher the interest rate the greater the cost of buying the underlying asset and carrying it to the future. Hence, the higher the interest rate, the greater the price of a call option.

6. **Dividend (D):** Benefits of ownership (i.e., dividends) accrue only to the holder of the asset and not to the holder of the derivative. When the dividends are paid, the value of the asset in the market (spot price) decreases by equivalent amount. With expected decline in the spot price the value of call must decrease and that of put must increase. It would be more beneficial to hold the stock than to hold the call.

The value of the call decreases with increased dividends. But the value of the put increases with increased dividend. This is because the holder of the put option retains the ownership of the asset till its exercise and all benefits keep accruing to the holder of the put option, because he is presumed to own the underlying asset.

Put-Call Parity Pricing Relationship

Put-call parity simply means parity in the values of European call and European put. It states that the value of a European call with a certain exercise price and expiry date is equal to the value of a European put with the same exercise price and expiry date. In other words, put-call parity relationship states that there is no arbitrage between the value of a European call and put options with the same strike price and expiry date on the same underlying asset (because their values are equal). Put-call parity is a financial relationship between the price of a put option and a call option on the same underlying asset with identical strike price and expiry. It explains the relationship between put, call, strike and bond prices. Put-Call Parity relationship was first identified by Hans Stoll in 1969. Put-call parity can be stated as:

$$C + PV(E) = P + S$$

where, C = Current market value of the call

PV (E) = Present value of the strike price E, discounted from the expiration date at a suitable risk free rate. It is expressed as e^{-rt} (r = rate of interest continuously compounded, t = time for expiration).

Thus, e^{-rt} represents present value at a certain percent of interest, i.e., discount (proportion of interest for the time to expiration).

P= Current market value of the put

S= Current market value of the underlying asset

The above equation sates that the value of a European call with an exercise price of E and expiry period of T plus the present value of the exercise price E equals the value of European put option with the same exercise price and date plus the value of the asset. The belief is that holding a stock and buying a put will provide the same pay-off as holding one call option and investing the present value of the exercise price. This equivalence is put call parity. To derive the put-call parity relationship, the assumption is that the options are not exercised before the date of maturity. Thus, the put-call parity is a concept related to European call and put options.

Put-call parity is an option pricing concept that requires the values of call and put options to be in equilibrium to prevent arbitrage. Whenever there is difference between put price and call price, arbitrage opportunities will prevail. If the put price is equal to call price, there will be no arbitrage opportunities.

Example

Consider two portfolios consisting of:

Portfolio A: Comprises one call option and an investment of the present value of the exercise price (in an investment in a zero coupon bond that matures to a value of E at the expiry of option), and

Portfolio B: Comprises one put option and one share.

Let us say the strike price for the put and call option is E.

After time T

Portfolio A: The deposit will mature and the holder will have ₹ E

If the stock price $S_T > E$, he will exercise the call using the deposit proceeds of ₹ E and in return he will receive a share that can be sold in the market for ₹ S_T.

If $S_T < E$, he will allow the option to expire and he will be left with the deposit proceeds of ₹ E.

Thus, at time T, Portfolio will be worth S_T or E, whichever is higher, i.e.,

Portfolio A = Max (S_T,E)

Portfolio B : At time T:

If $S_T > E$, the holder will allow the option to expire and hold the share with a market value of S_T. If $S_T < E$, the holder will exercise the put, selling the share at ₹ E.

Therefore, Portfolio B will be worth Min (S_T,E).

Therefore, irrespective of the share price, the two portfolios have the same terminal value. Hence, they must be of equal worth at T and the market will value them at the same price. Otherwise an investor could make an arbitrage profit by purchasing the less expensive portfolio, selling the more expensive one and holding the long-short position to expiration.

Value of Portfolio A at T is given below:

	Value at T	
At t = 0 (current value)	$S_T \leq E$	$S_T > E$
Buy one call	0	$S_T - E$
Invest PV (E)	E	E
Total (final value)	E	S_T

Value of Portfolio B at T is given below:

	Value at T	
At t = 0 (current value) =	$S_T \leq E$	$S_T > E$
Buy one share	S_T	S_T
Buy one put	$E - S_T$	0
Total (Final value)	E	S_T

Thus, both portfolios have the same end values.

So far we have assumed that the options that we are dealing with, are options on a stock that does not pay any dividend. We may now consider the question of dividends. Consider we have two portfolios-

Portfolio A: Comprises long call and an investment equivalent to the PV of the exercise price and the dividend expected to be received.

Portfolio B: Comprises a long stock and a long put.

Now at expiry, the portfolios will have one of the following as their values:

Portfolio A

	Value at expiry	
At t = 0 (current value)	$S_T \leq E$	$S_T > E$
Call	0	$S_T - E$
Investment	E + D	E + D
Final value (total)	E + D	$S_T + D$

Portfolio B

	Value at expiry	
At t = 0 (current value)	$S_T \leq E$	$S_T > E$
Long stock (pays a dividend)	$S_T + D$	$S_T + D$
Long put	$E - S_T$	0
Final value	E + D	$S_T + D$

Both portfolios have the same terminal value. Hence they will have a similar price to avoid arbitrage.

Put-Call Parity Relation (with Dividend)

The Put-Call Parity relation (with dividend) may be expressed as below :

C + PV (E) + PV (D) = P + S

Here, PV (D) = Present value of the dividend

The PV (D) may be calculated as below :

De^{-rt}

where, D = amount of dividend expected

e = exponential term, value is 2.7183

r = Risk free interest rate

t = time to receipt of dividend

From the above we can calculate the call price

C = P + S - PV(E) - PV (D)

where, C = Call price

P = Put price

S = Stock price

PV (E) = Present value exercise price

PV(D) = Present value of dividend expected

Calculation of Put Option Price and Call Option Price from Put-Call Parity Relationship

From the put-call parity, the put option price can be calculated easily. The equation for put call parity is:

$$C + PV(E) = P + S$$

From the above equation, the put option price P is calculated as below:

$$P = C + Ee^{-rt} - S$$

Similarly, the call option price C can be calculated as below:

$$C = P + S - Ee^{-rt}$$

Example 3

The current market price of a share is ₹ 155. The volatility of the share is measured as 20%. The risk free interest rate is currently 8% p.a. There is a call option and put option on the share. Time to expiration is 6 months with exercise price ₹ 150. Calculate put option price using put-call parity theory. The call price is ₹ 12.

Solution

$P = C + Ee^{-rt} - S$

C = ₹ 12

S = ₹ 155

E = ₹ 150

r = 8%

t = 6 months

$P = 12 + 150\ e^{-0.08 \times 6/12} - 155$

$P = 12 + 150 \times 2.7183^{-0.04} - 155$

$P = 12 + 150 \times 0.9608 - 155$

P = 12 + 144.12 - 155 = ₹ 1.12

Example 4

Calculate the call option price from the following data by using put-call parity theory:

Present value of Exercise price ₹ 128

Value of Put option ₹ 10

Current stock price ₹ 125

Solution

$C = P + S - Ee^{-rt}$

= 10 + 125 − 128

= ₹ 7

Importance or Uses of Put-Call Parity Relationship

The uses of put-call parity may be outlined as below :

1. Put-call parity principle links the put and call option prices. It tells us how to create synthetic instruments from traded ones.
2. Put-call parity relates prices of call, put, stock, and bonds. A bond can be replicated using stock, call and put. Likewise, the pay-off of stock can be obtained using bonds, call, and put.
3. Put-call parity demonstrates that given the price of a call, one can determine the price of a put.
4. Put-call parity also establishes a link between capital markets, derivative markets and debt markets.
5. Put-call parity can be used to check for arbitrage opportunities resulting from relative mispricing of calls and puts.
6. Put-call parity may be used to judge relative sensitivity to parameter changes, i.e., the difference in the reactions of calls and puts to changes in parameter values.

Boundary Conditions for Option Pricing (Upper and Lower Bounds for Options)

Before any attempt is made to find how much shall it cost to buy an option, we have to examine the boundary conditions (upper and lower bounds).

Call Option

A call option is a right to buy the asset at exercise price. Therefore, the maximum value (upper bound) cannot exceed the price of the asset itself. This is because no one would pay more money to buy a right to have the asset than what is actually required to buy the asset itself. Therefore, maximum value of the call option is:

$$C\ max = S \text{ or } C \leq S$$

where, C max = Maximum value of call

S = Spot price of the asset

Similarly, the minimum price that a call option would sell for depends upon its intrinsic value. If the option is maturing just at the time of buying, the time value of option is assumed to be zero and it must sell for its intrinsic value. However, if there is some time remaining for the maturity the exercise price would be payable only then. Therefore, the minimum value of the call option shall be the difference of spot price and the present value of exercise price. Thus,

$$C\ min = S - Ee^{-rt} \text{ or } C \geq S - Ee^{-rt}$$

where,

C min = Minimum value of call

Ee^{-rt} = Present value of the exercise price

t = time remaining for maturity

r = risk free rate of interest (discount rate)

Example 5

A share (stock) is selling for ₹ 500. The risk-free rate of interest is 10% p.a. continuously compounded. At what minimum price following call options on the share would sell for:

(a) A call with strike price of ₹ 450 maturing 1 month later.

(b) A call with strike price of ₹ 500 maturing 2 months later.

(c) A call with strike price of ₹ 550 maturing 3 months later.

Solution

The lower bound of call price is given as zero or $S - Ee^{-rt}$

Therefore, the minimum value of the call would be:

(a) $500 - 450 \times e^{-0.10 \times 1/12} = 500 - 446.23 =$ ₹ 53.77

(b) $500 - 500 \times e^{-0.10 \times 2/12} = 500 - 491.74 =$ ₹ 8.26

(c) $500 - 550 \times e^{-0.10 \times 3/12} = 500 - 536.42 =$ -₹ 36.42 (or zero, intrinsic value being negative)

Put Option

A put option is a right to sell the underlying asset at exercise price E. Therefore, the maximum value of put option cannot exceed the exercise price if it is exercised immediately. If there is still time remaining for maturity, the value of put cannot exceed the present value of the exercise price. Thus,

$$Pmax = Ee^{-rt} \text{ or } P \leq Ee^{-rt}$$

Intrinsic value is the amount of profit one would get if the option is exercised immediately. If it is paid at maturity, the difference of present value of exercise price and the spot price gives the lower bound. Therefore, the minimum price that a put option would sell for its intrinsic value. Thus,

$$P\ min = Ee^{-rt} - S \text{ or } P \geq Ee^{-rt}$$

Example 6

A share (stock) is selling for ₹ 500. The risk-free rate of interest is 10% p.a. continuously compounded. At what minimum price following put options on the share would sell for:

(a) A put with strike price of ₹ 450 maturing 1 month later.

(b) A put with strike price of ₹ 500 maturing 2 months later.

(c) A put with strike price of ₹ 550 maturing 3 months later.

Solution

The lower bound of put price is given by zero or $Ee^{-rt} - S$

Therefore, the minimum value of the puts would be:

(a) $450 \times e^{-0.10 \times 1/12} - 500 = 446.23 - 500 = -$ ₹ 53.77 (i.e., zero)

(b) $500 \times e^{-0.10 \times 2/12} - 500 = 491.74 - 500 = -$ ₹ 8.26 (i.e., zero)

(c) $550 \times e^{-0.10 \times 3/12} - 500 = 536.42 - 500 =$ ₹ 36.42

Option Pricing Models

Let us now move to the exact pricing or valuation of option so as to get the fair price (theoretical price or fair price). Determination of the fair price is important for both investors and trade` Investors want to know what the actual worth of the asset being acquired is. Traders want to identify the arbitrage opportunities.

Important Option Pricing Models

There are two basic models of option pricing. One is binomial option pricing model and the other is Black-Scholes option pricing model. In addition to these two models, there are some other methods such as Monte Carlo model, Black model, Finite difference model and Capital Asset Pricing Model. Binomial option pricing model is more of a computational procedure than a formula. Black-Scholes option pricing model is a mathematical formula. According to both methods we get the theoretical fair value of the option. Both models are used for pricing European options. The two models may be discussed in detail in the following pages:

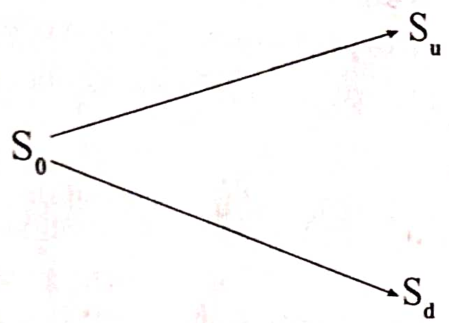
Binomial Option Pricing Model (BOPM)

The binomial model was developed by John Cox, Stephen Ross and Mark Rubinstein in 1979. Hence, this model is also known as C-R-R option pricing model. This model does not permit an analytical solution. It solves problems numerically. Under binomial model, we consider that the price of the underlying asset will either go up or go down in the period. Given the possible prices of the underlying asset and the strike price of an option, we can calculate the pay off of the option under these scenarios, then discount these pay off and find the value of that option as of today. Using these probabilities a tree would be constructed and evaluated to finally find the price of the option. This tree is called *binomial tree*. A binomial tree assumes that during a short interval of time, the stock will have only two possible values – an upside move or a downside move. However, over a longer period of time after several such moves, the stock (asset) can take a large number of distinct values.

The binomial model breaks down the time to expiration into potentially a very large number of time intervals or steps. A binomial tree of stock prices is initially produced working forward from the present to expiration. At each step it is assumed that the stock price will move up or down by an amount calculated using volatility and time to expiration. This produces

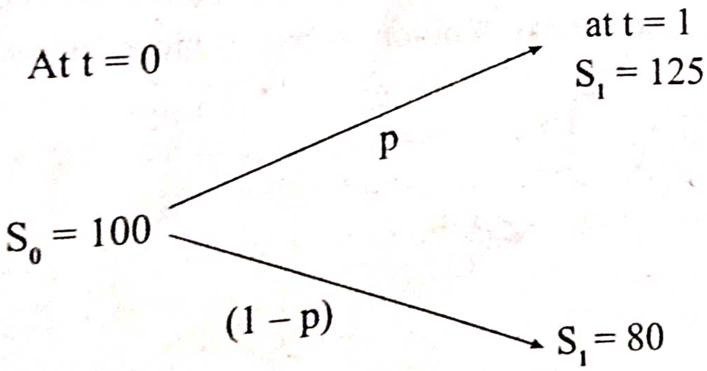
...nomial distribution (presented in the form of a tree) of underlying stock prices. The binomial ...e represents all the possible paths that the stock prices could take during the life of the ...tion. At the end of the tree, i.e., at the expiration of the option, the final possible stock ...ces are simply equal to their intrinsic values.

Binomial model is a discrete time model. It is based on binomial probability distribution. ...binomial probability distribution is a distribution in which there are two outcomes. The ...obability of up or down movement is governed by the binomial probability distribution. ...ecause of this, the model is also called a *two-state model*. In short, the price of underlying ...set is currently S_0 and it can take only two values S_u and S_d as shown below:



Possible Movements of Asset Price (Fig. 8)

To illustrate how the binomial model works, let us assume that a stock sells for a price ₹ 100 (S_0). We assume an investment horizon of one year and also assume that the price of the stock at the end of the investment period, S_1 (stock price at the end of one year) can either be 25% up, i.e., ₹ 125 or 20% down, i.e., ₹ 80. We also assume that stock pays no dividend and all returns are arrived through capital gains. The end position is shown as follows:



One Step (Single Period) Binomial Model (Fig. 9)

Let us assume that the probability of upward movement is P. The probability of downward movement then automatically becomes $1-p$.

Assuming this simplistic scenario, let us now attempt to value at-the-money European call with a strike price of ₹ 100 and time to expiry of one year. We know that the value of the call is Max(S – E, 0). A call with strike price at E = 100 means that if the price of the stock moves up to ₹ 120 the pay-off of the call option would be ₹ 20 (i.e., 120 - 100) and in case price moves down to ₹ 80, the call option expires worthless. Suppose the objective is to value a European call option to buy the stock for ₹ 110 in three months. If the stock price is ₹ 120, then the option price is ₹ 10 (i.e., 110 - 100). If the stock price turns out to be ₹ 80, then the option price will be zero (because the call option expires worthless).

One simple approach to value the call is to find the expected value of stock and find the expected value of call at t = 1. Then we discount the value at t = 0. To find the expected value of the underlying stock we need probabilities of both the branches of the binomial model. Assume that the probability of upward movement is p. Then the probability of downward movement is 1 - p. Suppose the probability of upward movement p is 0.60. Then the probability of downward movement (1 - p) is 0.40. Then expected value of stock at t = 1 will be 0.60 x 125 + 0.40 x 80 = ₹ 107.

Expected value of the call (at t = 1) = Max of (Expected value of stock - Exercise price) and zero = (107 - 100) or zero = ₹ 7.

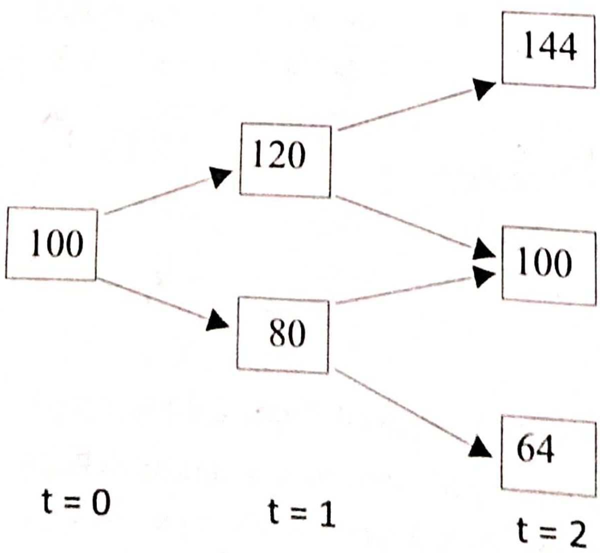
To find the value of the call today, we need to discount the end value at an appropriate rate. Suppose the appropriate rate of discount at risk free rate is 8%. Thus,

Value of call today = 7/1 + r = 7/1.08 = ₹ 6.48

When we use continuous compounding in calculation, value of call today = $7 \times e^{-rt} = 7 \times 2.7183^{-0.08 \times 1} = 7 \times 0.92312 =$ ₹ 6.46.

Binomial Tree

Initially a tree of stock prices is drawn showing the possible stock prices at every point of time in a forward form (left to right). This tree is known as binomial tree. It produces a binomial distribution of underlying stock prices. A binomial tree is given below:



Binomial Tree

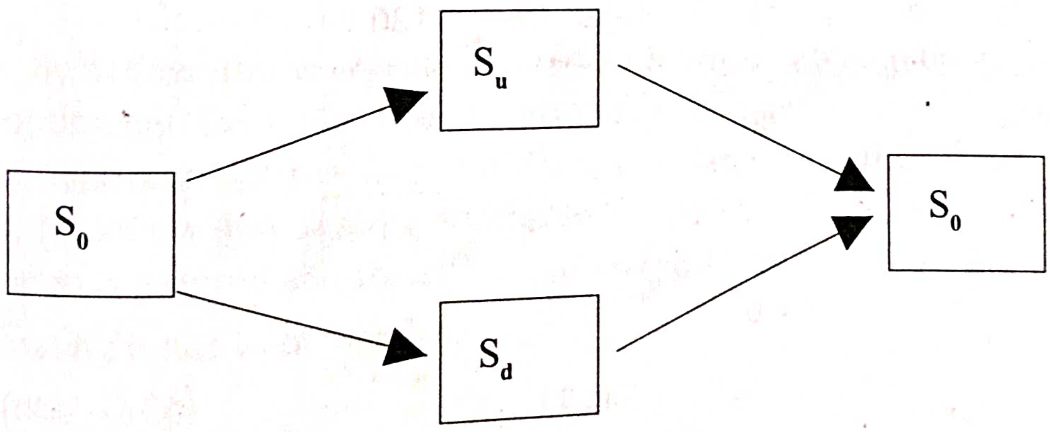
Fig. 10

This is a two period binomial tree. We can see that the stock price changes two times. Each point where two lines meet is called a *node*. This represents a possible future price of the stock. The tree is called a binomial because the spot price at every node can either move up or down. If we denote the stock price at the beginning as S_0 and S_u as the stock price in an up state and S_d as the stock price in a down state, then we can define the up factor as S_u/S_0 and down factor as S_d/S_0. The probability that the stock price will move from one node to another is known as transition probability.

Characteristics of Binomial Option Pricing Model

The following are the important characteristics of binomial option pricing model:

1. It is a discrete-time model.
2. Length of the time interval remains constant throughout the tree, i.e., the interval between the nodes is in months, it will be months everywhere, and if it is in terms of years, it will be years everywhere.
3. Volatility remains constant throughout the tree. Volatility is the variability about the mean value of the stock price.
4. The probability of an up movement and down movement remains the same in the entire tree.
5. The binomial tree is recombinant, i.e., an up move followed by a down move will take the stock to the same value as down move followed by an up move. Here the two paths merge or recombine.
6. Option price is determined by backward process calculation (i.e., working back from expiration to the present).



A recombining tree

Fig. 11

Assumptions of the Binomial Model

A binomial model is based on the following assumptions:

1. The current selling price of the stock (S) can take only two possible values, i.e., an upper value (S_u) and a lower value (S_d).

2. A perfect competitive market exists, i.e.,
   (a) There are no transaction costs, taxes or margin requirements.
   (b) The investors can lend or borrow at the risk-less rate of interest (r), which is the only interest rate prevailing.
   (c) The securities are tradable in fractions, i.e., they are divisible infinitely.
   (d) The interest rate (r) and the upswings/downswings in the stock prices are predictable.
3. The value of (1 + r) is greater than d, but smaller than u. This assumption ensures that there is no arbitrage opportunity.
4. The investors are prone to wealth maximization and lose no time in exploiting the arbitrage opportunities.
5. The price of security changes continuously.
6. The exercise price is E.

Binomial model will consider the time to expiry of an option as being one period, two-periods, and multiple periods.

Single Period Binomial Model

The single period binomial model is also known as *a one-step binomial model.* Here we shall assume a unit period of option's life. Unit period of option's life is implied that the option's stock price will move either up or down by the date of expiration of the option. We also assume that the stock is non-dividend. The binomial tree for single period is given below:

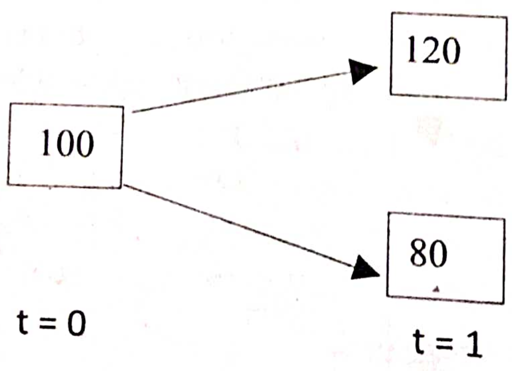


Fig. 11

There are two ways of calculating the value (or price) of a call option under binomial model. They are: 1. Option equivalent method, and 2. Risk neutral method.

Option Equivalent Method

The option equivalent method assumes that the outcomes for a particular share on which an option contract has been written can result into any two values, i.e., a high value or low value as compared to spot price for the call option. Accordingly, a hedge ratio can help in

Creating a portfolio with zero inflow and outflow to maintain the equilibrium price of a call option. Following steps are taken to calculate the price of call option.

Step 1: Estimate highest value of call assuming that the spot price of underlying asset on the expiration date will increase to a certain level in future.

$$C_u = S_u - E$$

Here,

C_u = Highest value of call

S_u = Maximum spot price of underlying share on expiration date as perceived by investor

E = Exercise price

Step II: Estimate lowest value of call assuming that spot price of underlying asset on the expiration date will decrease to a certain level in future.

Cd = Maximum (Sd – E, or zero)

Here,

Cd = Lowest value of call

Sd = Minimum spot price of underlying share on the expiration date as perceived by investor

E = Exercise price

Step III: Calculation of hedge ratio expressed as (h) or (Δ)

$$h \text{ or } \Delta = \frac{Cu - Cd}{Su - Sd}$$

Hedge Ratio: By hedge ratio we mean a ratio of difference between the highest and lowest value of call option and the difference of maximum and minimum price of underlying share. For example, hedge ratio 0.9 means an investor should buy 0.9 shares and sell one call option. By doing this, his inflow and outflow position will be neutral / indifferent. In this way risk of writing an option is covered and the seller of option is at no profit no loss.

Step IV: Estimating funds to be borrowed.

$$B = \frac{(dCu - uCd)}{(u - d) \times (1 + r)}$$

Here,

Cu = Highest value of call

Cd = Lowest value of call

d = lowest multiple of current spot price upto which spot price on the expiration date might decline.

Expected option value at C = (3 x 0.4742) + (0 x 0.5258) =

= 1.42 + 0 = 1.42

Note : For node C, the succeeding nodes are E and F. Option value at E is 3 and at F, it is zero.

Present value of expected option value at C

$= 1.42 \times e^{-rt} = 1.42 \times e^{-0.095 \times 1/12}$

= 1.42 x 0.9921 = 1.41

If the option holder will exercise his option at node B (in case of American option), then the value of the option will be max (160.45 - 130, 0) = 30.45. Here there are two values - binomial value 31.47 and exercise value 30.45. We take the maximum ₹ 31.47. Thus the value of American option at the early exercise point D = 31.47.

(iii) Option value at initial node A

Expected option value at A = (31.47 x 0.4742) + (1.41 x 0.5258)

= 14.92 + 0.74 = 15.66

Note : For node A, the succeeding nodes are B and C. Option value at B is 31.47 and at C, it is 1.41.

Present value of expected option value at A

$= 15.66 \times e^{-rt} = 15.66 \times e^{-0.095 \times 1/12}$

₹= 15.66 x 0.9921 = 15.54

Thus, the current value of call option (European) = ₹ 15.54

Note : The expected option value = Option up value x p + Option down value x 1 - p.

Thus, the option value is the weighted average of the option's two possible values (up and down).

Note : In the same manner we can find the values at each node in the three period binomial option pricing model.

Multi-period Binomial Model

The binomial model can be extended to periods beyond two to any number of time periods. This is called multi-period binomial model. We can find binomial option prices for any number of periods. The steps for the calculation of option price under multi-period binomial model are as follows:

1. Build a price tree (binomial tree) for the underlying asset.
2. Calculate the possible option values in the last period (time T = expiration date)
3. Set up all possible risk-less portfolios in the penultimate (last but one) period.
4. Calculate all possible option prices in the penultimate period.

5. Keep working back through the tree to "Today" (time T-n in an n- period, (n+1) –date, model).

The general formula, with n periods remaining to maturity until the option expires and no dividends on the European call price is given below:

$$C = \frac{1}{(1+r)^n} \sum_{i=0}^{n} \frac{n!}{i!(n-i)!} p^i (1-p)^{n-i} \mathrm{Max}(0, Su^i d^{n-i} - K)$$

Advantages or Uses of Binomial Option Model

The following are the advantages of binomial option pricing model :

1. Binomial option pricing model is widely used as it is able to handle a variety of conditions for which other models cannot easily be applied.
2. This model uses simple mathematical calculations for stock prices as well as option prices.
3. This is considered to be more accurate, particularly for longer-dated options, and options on securities with dividend payments.
4. It is easy to calculate the option price with a computer spread sheet.
5. Binomial option model can be used to price accurately the American option. This is because with the binomial model, it is possible to check at every point in an option's life, (i.e., at every step of the binomial tree) for the possibility of early exercise.
6. It explains the procedure as to how the construction of a riskless hedge leads to finding out the option price.
7. This is a better model for valuing employee stock option, interest rate option and currency option.

Limitations of Binomial Option Pricing Model

There are few limitations to binomial option pricing model. They are :

1. The basic assumption that there are only two possibilities for share price over next one period is impractical and hypothetical.
2. This model is relatively slow.
3. This model is not a practical solution for the valuation of thousands of prices quickly.
4. It is much more complex than the Black -Scholes model.

The Black and Scholes Option Pricing Model

In 1973, Fisher Black and Myron Scholes developed a model for pricing of options. They published the path-breaking paper titled, "The Pricing of Options and Corporate Liabilities". Today, the model is known as the Black-Scholes model. Myron Scholes and Robert Merton won the Nobel Prize in economics for their contributions in option pricing in 1997. Black had died in 1995, but otherwise would have shared the prize.

Black-Scholes model has become the standard method of pricing options. This is a model used to calculate the value of a European call option. The model utilises the stock price, strike price, expiration date, risk free return and the standard deviation (volatility) of the stock's return. The model can be arrived using complex calculus and differential equations. Alternatively, the Black-Scholes model can be also be arrived with risk neutral valuation approach.

The Black-Scholes formula for calculating the price of a call option is:

$$C = SN(d_1) - E\,(e^{-rt})\,N(d_2)$$

where,

C = Theoretical price of the call option

S = Current stock price

N = Cumulative normal distribution

E = Exercise price of the option

e = exponential term, its value is 2.7183

r = risk free interest rate

t = time to expiration (i.e., time remaining before the expiration).

The calculation of option price through the Black-Scholes formula involves many intermediary computations. Hence, a systematic procedure is required. The procedure is as follows:

1. It is better to start with the calculation of d_1. It is calculated as follows:

$$d_1 = \frac{\ln\left(\frac{S}{E}\right) + \left(r + \frac{\sigma^2}{2}\right) \times t}{\sigma\sqrt{t}}$$

Here, ln = Natural logarithm of the bracket number.

σ = Annual volatility of the stock.

2. Then calculate d_2. This is calculated (from d_1) as follows:

$$d_2 = d_1 - \sigma\sqrt{t}$$

3. Find cumulative normal distribution values using N (d) tables. Thus, $N(d_1)$ and $N(d_2)$ are the values of cumulative normal distribution at d_1 and d_2.

4. Now calculate the price of the option by substituting the respective values by using the first formula.

It may be noted that if 't' is in years, then σ and r should also be in annual terms.

Assumptions of Black-Scholes Model

The assumptions underlying the Black-Scholes Model are as follows:

1. The risk free interest rate is constant.
2. There are no transaction costs.
3. There are no taxes.
4. There are no riskless arbitrage opportunities; security trading is continuous.
5. There is no dividend payment on shares.
6. Short selling is permitted.
7. The volatility of the underlying instrument is known and constant.
8. The distribution of the possible share prices (or index levels) at the end of a period of time are log normal. In other words, a share's continuously compounded rate of return follows a normal distribution (i.e., returns on the underlying stock are normally distributed).
9. The market is an efficient one. The people cannot predict the direction of the market or any individual stock.
10. The option being valued is a European style option, with no possibility of an early exercise (i.e., exercise only on maturity).

Volatility

Volatility is a measure of the variability in the prices of securities or assets. It is estimated by using a statistical measure, i.e., standard deviation (σ). There are two forms or types of volatility – historical volatility and implied volatility. Historical volatility is calculated from the historical stock prices. We can find the option price when a volatility (historical) is given. But a question arises here : What is the volatility for the observed option price to be consistent with the Black – Scholes formula ? The answer is the *implied volatility*. Thus, when the volatility of the stock implies the current option price, we consider it is the implied volatility.

Example 12

The current price of a stock is ₹ 90 per share. The risk free interest rate is 8% (annualised continuous compounding). If the volatility of the stock is 23% p.a., what is the price of the ₹80 call option expiring in 6 months (according to Black and Scholes model)?

Solution

$$C = S\,N(d_1) - Ee^{-rt}N(d_2)$$

Step 1 Calculation of d_1

$$d_1 = \frac{\ln(S/E) + (r + \sigma^2/2)\times t}{\sigma\sqrt{t}} = \frac{\ln(90/80) + (0.08 + 0.23^2/2)\times 6/12}{0.23\sqrt{6/12}}$$

$= 1.0517$

Note: $\sigma^2/2$ can be taken as $\sigma^2 \times 0.5$

Step 2 Calculation of d_2

$$d_2 = d_1 - \sigma\sqrt{t} = 1.0517 - 0.1626 = 0.8891$$

Critical Evaluation of Black and Scholes Model

Black and Scholes model is being criticised on several grounds. The following are the limitations of the model:

1. It does not yield satisfactory results for options that are deep in-the-money or out-of-the money.
2. The model does not yield unbiased values in respect of stocks with very high or very low volatility and mispricing increases, as the time until expiration increases.
3. It cannot be used to accurately price options on American style. It only calculates the option price at one point of time, i.e., at expiration. However, empirical studies indicate that the Black-Scholes model applies to American options as well.
4. It cannot accurately calculate the theoretical value of a dividend paying stock option.

Despite these limitations, the model does a reasonable job in pricing a variety of derivative instruments. But the real utility of the model is that it provides us a mechanism to hedge an option and the cost of hedging gives us insights into the likely price of the option. In the Black-Scholes model, all the data inputs are directly observable except volatility. The main advantage of Black-Scholes model is speed. We can calculate a very large number of option prices in a very short time. B-S model is very popular and it is useful to option holder to know whether the option is over-priced or under-priced. The model provides an excellent analysis in valuation of debt relation to equity. In short, the B-S model gives a mathematical formula for calculating the theoretical prices of both call option and put option.

Difference between Binomial Model and Black-Scholes Model

Binomial model and B-S model are the two important option pricing models. There are some basic differences between these two models. Binomial model uses a computational procedure (numerical method). But B-S model uses an analytical approach. Binomial model is a flexible model to value options that are not regularly traded or are specifically negotiated between contracting parties. B-S model is an analytical model that values conventional options that are regularly traded.

The Binomial model assumes that percentage change in share price follows a binomial distribution, while the B-S model is based on the assumption that it follows a log normal distribution.

Black- Scholes model was first developed for European option. But binomial option pricing model is very suitable for American style options.

Binomial model is a discrete-time model with specific time intervals. The Black-Scholes model, however, takes into account an infinite number of sub-intervals, on a continuous time basis.

Relationship of Binomial Model with Black Scholes Model

As the number of intervals is increased, the values we obtain from the binomial model shall get closer and closer to the Black-Scholes value. In other words, the binomial model tends to move towards the Black-Scholes Model as the number of time intervals (in the price movement) increases. The mathematical proof for such convergence was discussed by Jarrow and Rudd (1983), Cox, Ross and Rubinstein (in 1979) and Rendlemen and Bartter (in 1979). In fact, we can think of the Black-Scholes formula as a shortcut alternative to the binomial model as the number of intervals gets very large.

Option Greeks

We have seen that option price depends upon five facto` These factors include asset price (So), exercise price (E), time remaining for expiration (t), the risk free rate of return (r), and volatility measured by standard deviation (s). All these factors are determinants of option price. Whenever, these factors change (except exercise price), the option price will change. Thus option value is sensitive to these factors or variables. The variations in the option value with respect to each determinant of price are denoted by Greek letters such as delta, gamma, theta, rho, vega (vega is not a Greek letter). Hence they are also referred to as option Greeks. Following are the Greeks of options:

Greek Letter	*Variable/Determinant of Option Price*
Delta	Change of asset price
Theta	Time left for maturity
Gamma	Change in delta (change in price)
Rho	Change in risk free rate
Vega	Change in volatility

Option Greeks may be briefly outlined as below:

Delta: Option delta is the sensitivity of an option price relative to change in the price of the underlying asset. In short, it is the change in the option value when the stock price changes by 1. It is denoted by Δ. It is calculated as below:

$$\text{Option delta} = \frac{\text{Change in option value}}{\text{Change in stock or asset price}} = \frac{\text{Cu - Cd}}{\text{Su - Sd}}$$

Theta: Theta measures the sensitivity of option price in relation to its time to expiration. It tells us how much value the option would lose after one day with all the other variables remaining the same. Theta is always negative for the buyer of the option because the value of the option goes down each day if his view is not realised. It is positive for the seller of the option.

Gamma: Gamma is the rate of change of delta of the option or of the portfolio of options. It is the partial derivative of delta with respect to price. It tells us how much the delta, if an option would increase or decrease for a unit change in the price of the underlying.

Vega: Vega is the change of value of the option if the volatility changes. It indicates how much the option premium would change for a unit change in annual volatility of the underlying.

Vega is positive for calls and puts. In fact, it is the volatility that drives the prices of the call and put option.

Rho: This is the rate of change in the value of option of premium to the domestic interest. It gives the rate of change of option price with respect to risk free interest rate. Value of rho is till positive for call and negative for put option.

PRACTICAL PROBLEMS

Illustration 1

The share of X Ltd. stands at ₹ 120, put options with a strike price of ₹ 130 are priced at ₹ 15.

a) What is the intrinsic value of options ?

b) What is the time value of options ?

c) If the share price falls to ₹ 60 by the expiry date, what would be the profit/loss for the holder and writer of the options ?

Solution

a) The intrinsic value = Strike price - Stock price

=₹ 130 - ₹ 120 = ₹ 10

b) Time vale = Premium - Intrinsic value

=₹ 15 - ₹ 10 = ₹ 5

c) The option holder could exercise to realize the intrinsic value of ₹ 70 (i.e., 130 - 60). When the payment (premium) of ₹ 15 considered, net profit is ₹ 70 - 15 = ₹ 55. The option writer would pay ₹ 130 for a share worth ₹ 60. This loss of ₹ 70 is partially offset by the premium receipt of ₹ 15. Net loss will be ₹ 70 - 15 = ₹ 55. So profit of the buyer is equal to the loss of the writer.

Illustration 2

Calculate lower bound from the following data :

Stock price : ₹ 270 per share

Style of option : European

Type of option : Call

Strike price ₹ 265 per share

Interest rate = 10% p.a.

Time to expiration = 6 months

Dividend : Nil

Solution

A lower bound for the price of a European call option on a non-dividend paying is calculated by the following formula :

$S - Ee^{-rt}$